TRINITY COLLEGE

Year 12 Mathematics Applications
Test 5 2016

Section 1 Calculator Free

Finance II, Spanning Trees, Maximum Flow

STUDENT'S NAME Solutions

DATE: Friday 9th August **TIME:** 30 minutes **MARKS**: 30

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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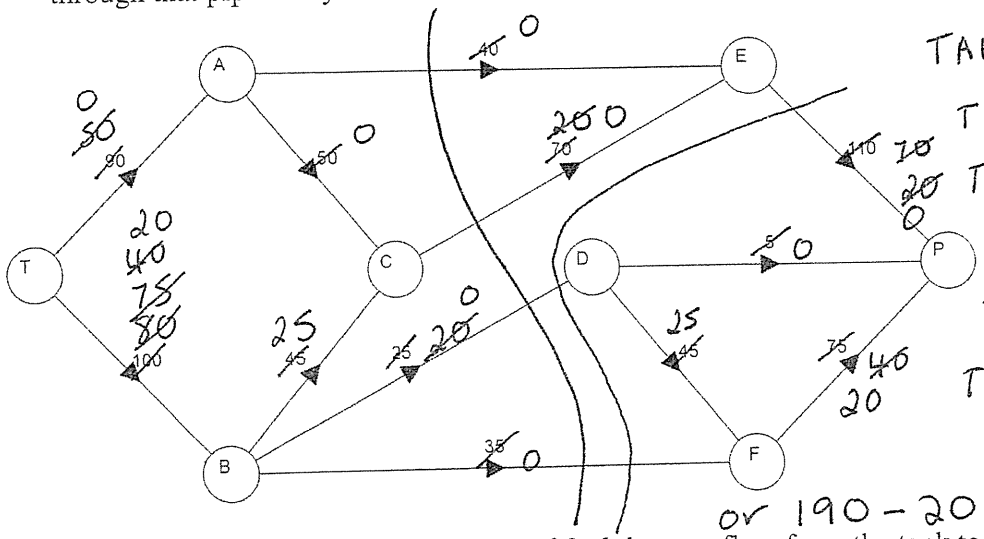
1. (8 marks)

Fuel can flow from a tank at T to a power station at P through a network of pipes as shown below. The number on each arc represents the maximum amount of fuel, in kL, that can flow through that pipe every hour.



TAEP - 40
TACEP - 50
TBCEP - 20
TBDP - 5
TBFP - 35
TBDFP - 20
170 kL

or 190 − 20 = 170 kL

(a) What is the maximum amount of fuel that can flow from the tank to the power station every hour? Show use of systematic working. [4]

2 marks crossing on diagram 1 in total or diff 1m 170 kL

(b) Confirm your answer to part (a) by drawing a cut of the same value on the network. [2]

✓✓

More than one correct answer

(c) An engineer has suggested building an additional pipe from C to D with a maximum flow rate of 30 kL per hour. What effect, if any, would this have on your answer to (a)? [2]

Flow increased to 190 kL ✓✓

or Flow increased by 20 kL

2. (8 marks)

The table below shows the distance in km, between five towns. A blank entry indicates that these two towns are not directly linked.

	A	B	C	D	E
A	-	12	18	-	27
B	12	-	15	11	16
C	18	15	-	13	9
D	-	11	13	-	14
E	27	16	9	14	-

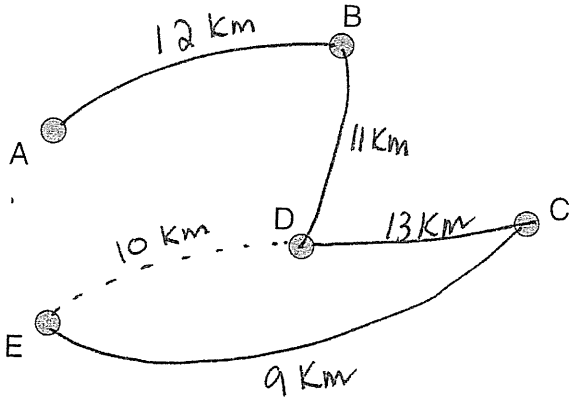
(a) Use Prim's algorithm to determine the minimum spanning tree for this network. [3]
State the connections that form the minimum spanning tree.

AB, BD, CE and CD

1m table
2m connections listed

[2]

(b) Draw the minimum spanning tree.



✓✓
OK if not km

(c) What is the minimum amount of road required so that it is possible to travel from any one town to any other? [1]

45 km ✓

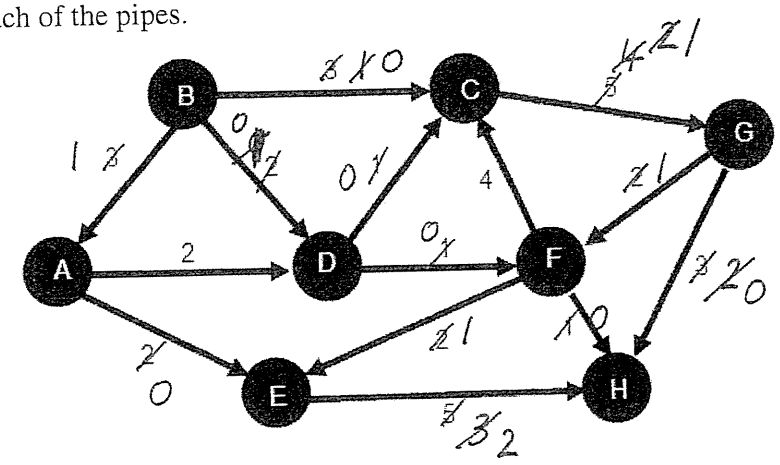
(d) A new road of 10 km is to be constructed between towns E and D. What effect, if any, will this have on the minimum spanning tree? [2]

CD replaced by ED ✓
new distance of 42 Km ✓
(reduced by 3 Km)

must state new tree as well as effect on distance for 2m

3. (6 marks)

The following diagram illustrates a drainage system. The numbers indicate the flow rate (L/min) of each of the pipes.



(a) State which node is the:

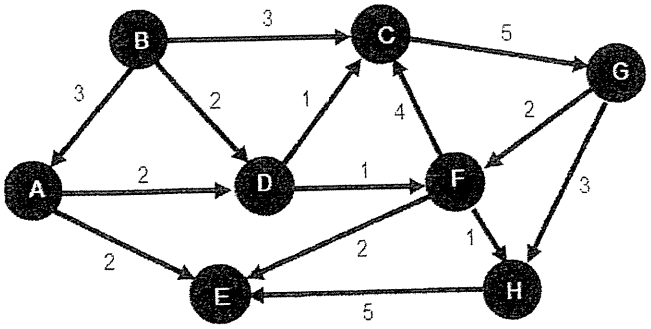
(i) source B ✓ [1]

ii) sink H ✓ [1]

(b) Calculate the maximum flow for the system. Show working on the diagram above, stating your result here. [3]

diagram ✓ 9 − 2 = 7 L/min ✓ ✓

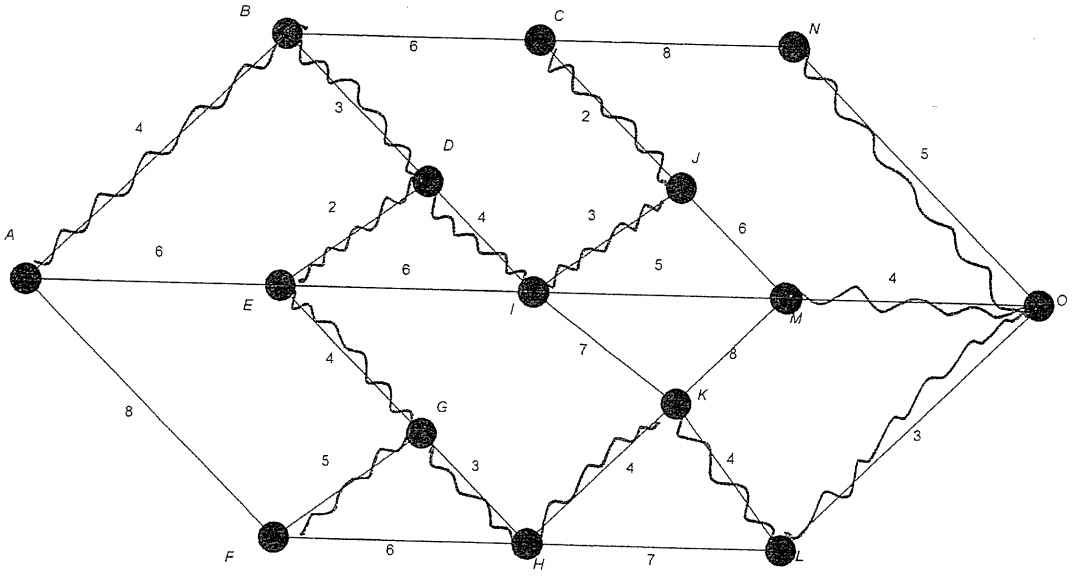
(c) If the direction of flow between nodes E and H is reversed (as shown below), what significant change does this have on the system? [1]



sink is now E ✓

4. (8 marks)

A severe storm has forced Valentine Power Supply Company to shut down part of its network supplying power to a number of the suburbs of Njoo City. Valentine Power Supply Company technicians have assessed the damage and have determined the affected areas can be represented by the following network consisting of 15 nodes A, B, …. ,O.



The edges of the network represent power supply lines that require repair. The number on each such edge represents the time in hours required to repair the line. Once a line is repaired, power is restored to the nodes it connects. A first objective of Valentine Power is to restore power to the affected areas in the minimum time possible.

(a) Indicate on the diagram above the minimal spanning tree for the network. [4]

-1 per error

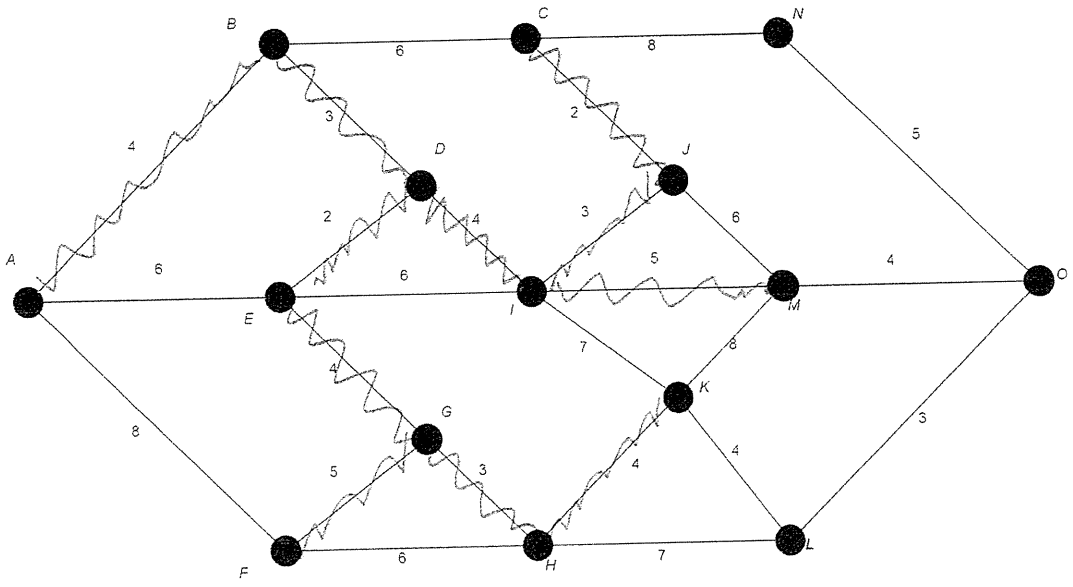
(b) What is the minimum time required to restore power to the areas that lost power? [1]

4 + 3 + 2 + 4 + 5 + 3 + 4 + 3 + 2 + 4

+ 4 + 3 + 4 + 5

= 50 hours

(c) In response to customer dissatisfaction, Valentine Power decides to engage an external crew to deal with areas L, N and O, and focus its resources on the remaining part of the network. What now is the minimum time required to restore power to the network? [3]



Remove 5+4+3+4 = 16 ✓

add 5 (I–M) ✓

∴ Time decreased by 11

∴ Min time is 39 hours. ✓

TRINITY COLLEGE

Year 12 Mathematics Applications
Test 5 2016

Section 2 Calculator Assumed

Finance II, Maximum Flow, Minimum Spanning Tree

STUDENT'S NAME Solutions

DATE: Friday 12th August **TIME:** 20 minutes **MARKS**: 22

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (4 marks)

An old boy wishes to leave an annual perpetual award to the College. What initial investment is required if the account pays annual interest of 5%, compounded monthly and the award is to be $12 000 per year?

Payment = Interest $= x\left(1+\frac{0.05}{12}\right)^{12} - x$ ✓

solve $x\left(1+\frac{0.05}{12}\right)^{12} = 12000 + x$ ✓✓

$\therefore$ Investment is \$234 549.55 ✓

OR

Could calculate effective interest rate is 0.05116 ✓

$Perp = \frac{payment}{rate}$ ✓

$= \frac{12000}{0.05116} = \$234\,549.55$ ✓✓

*If do $\frac{12000}{0.05} = \$240000$ give 1 mark

6. (11 marks)

James has \$750 000 in an annuity that pays interest at 2.5% on the opening annual balance. Interest is paid at the end of the year. From the end of the first year on, James withdraws \$45 000 at the end of each year after the interest has been paid. The table below show the annual account balance for the first few years.

Year	Opening balance	Interest	Annual Withdrawal	Closing balance
1	\$750 000	\$18 750	\$45 000	\$723 750
2	\$723 750	\$18 093	\$45 000	\$696 843.75
3	\$696 843.75	\$17 421.09	\$45 000	\$669 264.84
4	\$669 264.84	\$16 731.62	\$45 000	\$640 996.46
5	\$640 996.46	\$16024.91	\$45000	\$612021.38

.37 ok

(a) Complete the table for year 5. [2]

(b) Write a recursion rule for the closing balance for each year. [2]

$T_{n+1} = T_n \times 1.025 - 45\,000$, $T_0 = 750\,000$

(c) Determine the account balance at the end of 15 years. [2]

\$279 286.93

(d) How many years will the fund last? [2]

if give 21.83 /m if 21 give /m

22 years

(e) If instead James wishes for the account balance to stay stable, how much should he withdraw each year? [1]

PV = FV

\$18 750

(f) If instead James wishes the account to last 20 years, what annual withdrawal should he make? [2]

\$48 110.35

N = 20
I% = 2.5
PV = -750000
PMT = ?
FV = 0
P/Y = 1
C/Y = 1

7. (3 marks)

A person wishes to deposit \$320 000 into an account that will earn interest of 7.5% per annum, compounded annually, withdrawing \$22 000 at the end of the first year, \$22 660 at the end of the second year, \$23 339.80 at the end of the third year, and so on, with each annual withdrawal being a 3% increase on the year before.

Write a recursive rule for this situation.

$T_{n+1} = T_n \times 1.075 - 22000 \times 1.03^{(n-1)}, \quad T_0 = 320\,000$ ✓ ✓ ✓

OR

$T_{n+1} = T_n \times 1.075 - B_n, \quad T_0 = 320\,000$

$B_{n+1} = B_n \times 1.03, \quad B_0 = 22000$

8. (4 marks)

Horatio deposits \$475 000 into an annuity which earns 7.5% interest per annum, compounded monthly. The annuity pays an annual payment of \$55 000.

(a) How many payments of \$55 000 will be made? [2]

$N = ?$
$I\% = 7.5$
$PV = -475000$
$PMT = 55000$
$FV = 0$
$P/Y = 1$
$C/Y = 12$

$N = 14.85$ ✓

$\therefore$ 14 payments ✓

(b) What will be the value of the final payment? [2]

When $N = 14$, $FV = \$43\,479.47$ ✓

$\therefore$ Payment is $43479.47 \times \left(1 + \frac{0.075}{12}\right)^{12}$

$= \$46\,854.89$ ✓